TARGET MATHEMATICS
THE EXCELLENCE KEY
AGYAT GUPTA (M.Sc., M.Phil.)

21st YEAR OF QUALITY EDUCATION

CODE:- AG-TS-9-5544 पंजियन क्रमांक REGNO:-TMC -D/79/89/36

## GENERAL INSTRUCTIONS :-

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 4 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

**सामान्य निर्देश :**

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 4 हैं।
7. प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें।

# PRE-BOARD EXAMINATION 2012 -13

Time : 3 Hours — अधिकतम समय : 3
Maximum Marks : 100 — अधिकतम अंक : 100
Total No. Of Pages :4 — कुल पृष्ठों की संख्या : 4

**CLASS – XII** **CBSE** **MATHEMATICS**

## SECTION A

**Q.1** Check whether the relation R in R defined by R = $\{(a,b): a \le b^2\}$ is transitive. Ans = not $(4,-3)\in R \,\&\, (-3,1)\in R \Rightarrow (4,1)\notin R$

**Q.2** Evaluate : $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos \alpha} dx$.. Ans : $2\sin x + 2x\cos\alpha$

**Q.3** $A = \begin{bmatrix} 1 & 0 & 1 \\ k & 1 & 2 \\ 0 & 1 & 4 \end{bmatrix}$, then write $|3A|$ and find the value of k, if A is singular matrix. ans : 0 ( zero)

**Q.4** The income I of Dr. Rastogi is given by $I(x) = Rs.\ (x^3 - 3x^2 + 5x)$. Can an insurance agent ensure him for the growth of his income? Ans : Show that $\frac{d}{dx} I(x) > 0$ for all x.

**Q.5** Find the value of x if the area of $\Delta$is 35 square cms with vertices (x,4 ),(2,-6)and (5,4 ). Ans x = -2 or 12

**Q.6** Evaluate : $\int [1 + 2\tan x(\tan x + \sec x)]^{1/2} dx$. Ans $\log(\sec x + \tan x) + \log \sec$

**Q.7** The contentment obtained after eating x-units of a new dish at a trial function is given by the Function $C(x) = x^3 + 6x^2 + 5x + 3$ . If the marginal contentment is defined as rate of change of (x) with respect to the number of units consumed at an instant, then



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| | |
|---|---|
| | find the marginal contentment when three units of dish are consumed . ANS : **68 units** |
| **Q.8** | If $\vec{a}$ *and* $\vec{b}$ are non-collinear vectors, find the value of x for which the vectors $\vec{I}=(2x+1)\vec{a}-\vec{b}$ *and* $\vec{m}=(x-2)\vec{a}+\vec{b}$ are collinear. Ans x = 1/3 |
| **Q.9** | If $\vec{a}=\vec{b}+\vec{c}$ , then is it true that $\lvert\vec{a}\rvert=\lvert\vec{b}\rvert+\lvert\vec{c}\rvert$ ? Justify your answer. Ans = not |
| **Q.10** | Find the perpendicular distance from ( 2,5,6) on XY plane . Ans : 6 unit |
| | **SECTION B** |
| **Q.11** | Solve the following equation : $3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$. Ans $6\tan^{-1}x-8\tan^{-1}x+4\tan^{-1}x=\frac{\pi}{3}\because 2\tan^{-1}x=\frac{\pi}{3}\Rightarrow\tan^{-1}x=\frac{\pi}{6}\therefore x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$ <br> OR <br> Solve for x : $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x), 0<x<\frac{\pi}{2}$. Ans $x=\frac{\pi}{4}\in\left(0,\frac{\pi}{2}\right)$ |
| **Q.12** | If f(x ) and g (x) be two invertible function defined as $f(x)=\frac{2x+1}{3x-5}$ be defined as $g(x)=\frac{3x+3}{7x-2}$. Prove that $(gof)^{-1}=f^{-1}og^{-1}$. Ans : $(gof)x=\frac{15x-12}{8x+17}\Rightarrow(gof)^{-1}=\frac{12+17y}{15-8y}$ <br> $f^{-1}=\frac{1+5x}{3x-2}\ \&\ g^{-1}=\frac{2x+3}{7x-3}\Rightarrow f^{-1}og^{-1}=\frac{12+7y}{15-8y}$ |
| **Q.13** | Using the properties of determinants, prove the following: $\begin{vmatrix}1+a^2-b^2 & 2ab & -2b\\ 2ab & 1-a^2+b^2 & 2a\\ 2b & -2a & 1-a^2-b^2\end{vmatrix}=(1+a^2+b^2)^3$ . |
| **Q.14** | An air force plane is ascending vertically at the rate of 100 km/ h. If the radius of the earth is r km, how fast is the area of the earth , visible from the plane , increasing at 3 minutes after it started ascending ? Given that the visible area A at height h is given by $A=\frac{2\pi r^2h}{r+h}$. Ans $\frac{dA}{dt}=\frac{200\pi r^3}{(r+5)^2}$ |
| **Q.15** | If $y=\sin(m\sin^{-1}x)$ , prove that $(1-x^2)\frac{d^2y}{dx^2}-x\frac{dy}{dx}+m^2y=0$. <br> OR <br> If $x^y=e^{x-y}$ , prove that $\frac{dy}{dx}=\frac{\log x}{(1+\log x)^2}$ . |
| **Q.16** | Two airships are moving in space along the following lines $\frac{x-3}{1}=\frac{5-y}{2}=\frac{z-7}{1}$ and $\frac{x+1}{7}=\frac{y+1}{-6}=\frac{z+1}{1}$. An astronaut wants to move from one ship to another ship when two airships are closest. What is the least distance between the ships that he has to travel? ans :Find S.D. $\Rightarrow 2\sqrt{29}$ units. |
| **Q.17** | Find all the local maximum values and local minimum values of the function $f(x)=\sin 2x-x, -\frac{\pi}{2}<x<\frac{\pi}{2}$. Ans $f'(x)=0\therefore x=\pm\frac{\pi}{6}$. f(x ) is maximum at $x=\frac{\pi}{6}$ and maximum value is $f\left(\frac{\pi}{6}\right)=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$ and f(x) is minmum at $x=-\frac{\pi}{6}$ & minmum value is $f\left(-\frac{\pi}{6}\right)=-\frac{\sqrt{3}}{2}+\frac{\pi}{6}$ |
| **Q.18** | Evaluate : $\int\frac{x}{x^3-1}dx$ . Ans. $\frac{1}{3}\log(3x-1)-\frac{1}{6}\log(x^2+x+1)+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$ <br> OR <br> Evaluate $\int\frac{\sin 4x-2}{1-\cos 4x}e^{2x}dx$. Ans $\frac{1}{2}e^{2x}\cot 2x$ |
| **Q.19** | Solve the differential equation, $(1+y+x^2y)dx+(x+x^3)dy=0$ where y = o when x = 1 |



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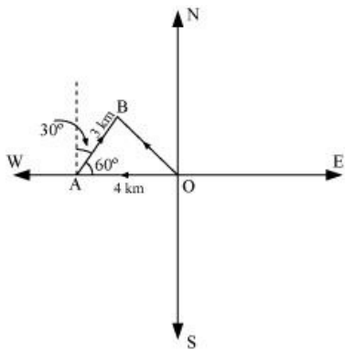
| | |
|---|---|
| | Ans. $yx = -\tan^{-1} x + \frac{\pi}{4}$ |
| **Q.20** | A girl walks 5 km towards west, and then she walks 3 km in a direction 60° east of north and stops. Determine the girl's displacement from her initial point of departure. *Respect the girl implies respect the nation comment on it.* ***ANS :*** Let O and B be the initial and final positions of the girl respectively. Then, the girl's position can be shown as:<br>  <br><br>Now, we have:<br>$\overrightarrow{OA} = -4\hat{i}$<br>$\overrightarrow{AB} = \hat{i}\left\lvert\overrightarrow{AB}\right\rvert\cos 60° + \hat{j}\left\lvert\overrightarrow{AB}\right\rvert\sin 60°$<br>$= \hat{i}3\times\frac{1}{2} + \hat{j}3\times\frac{\sqrt{3}}{2}$<br>$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$<br>$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$<br>$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$<br>$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$<br>$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$<br>$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$<br>Hence, the girl's displacement from her initial point of departure is $\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$.<br>*Respect the girls is the respect of the nation because if we respect the girls and ladies so we will respect the mother or sister or wife of someone's and this respect will lead the nation from hate speeches and battle we should never forget Mahabharata Battle and Demolishing of Ravan is just because Ladies Dropadi and Seeta*<br>OR<br>If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ & $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$, find a unit vector which is linear combination of $\vec{a}$ & $\vec{b}$ and is also perpendicular to $\vec{a}$. "Directionless youth is the burden on nation" comment on it. ans : *Directionless student/youth is just like a stone on road any one can misguide him or her and drive them as per their own will, as they don't have any ambition or any aim moreover they always frustrated from the world and such type of people are the burden on society and the country.* Ans. $= \frac{-(5i + 4j + k)}{\sqrt{42}}$ |
| **Q.21** | Form the differential equation of the family of curve $y = ae^{x} + be^{2x} + ce^{3x}$; where a, b, c are some arbitrary constants. Ans. $y_3 - 6y_2 + 11y_1 - 6y = 0$ |
| **Q.22** | The compressor used in refrigrtetors are manufactured by three different factories at Pune, Nasik and Nagpur. It is known that the Pune factory produces twice as many compressors as the Nasik one, which produces the same number as the Nagpur one ( during the same period ). Experience also shows that 0.2 % of the compressors produced at Pune as well as at Nasik are defective and so are 0.4 % of those produced at Nagpur. A quality controller chooses a compressor and finds at a defective one. What is the probability that it was produced at Nasik factory? Ans 1/5 |
| | **SECTION C** |
| Q.23 | Three friends A, B and C visited a Super Market for purchasing fresh fruits. A |



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purchased 1 kg apples, 3 kg grapes and 4 kg oranges and paid Rs. 800. B purchased 2 kg apples, 1 kg grapes and 2 kg orange and paid Rs. 500. White C paid Rs. 700 for 5 kg apples, 1 kg grapes and 1 kg oranges. Find the cost of each fruit per kg by using matrix method. Why are the fruits good for health?

ans Let cost of each fruit be x, y, z respectively. Then solve the equations so formed by using matrix method. So, x = 100, y = 100, z = 100. Hence the cost of each fruit is ₹100 per kg. **Importance of fruits:** Fruits contain nutrients and vitamins which help our body in its proper growth and maintenance.

OR

Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} p \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$. Ans

$P=\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

Q.24 Reduce in symmetrical form , the equation of the line x – y + 2z = 5 , 3x+ y + z = 6 . ans : d . r . of line is – 3 , 5 , 4 & point $\left(\frac{11}{4},-\frac{9}{4},0\right)$. Equation of required line

$$\frac{x-\frac{11}{4}}{-3}=\frac{y+\frac{9}{4}}{5}=\frac{z-0}{4} \qquad eq\,of\,line\ \frac{4x-11}{-3}=\frac{4y+9}{5}=\frac{z}{1}$$

Q.25 Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga? *Why Meditation and Yoga is necessary and sufficient thing for peace in mind and for good health.* Ans.

$= P(E_1)=P(E_2)=\frac{1}{2}; P(A/E_1)=\frac{40}{100}\times(1-\frac{30}{100})=\frac{28}{100}; P(A/E_2)=\frac{40}{100}\times(1-\frac{25}{100})=\frac{30}{100}$

$$= \frac{\frac{1}{2}\times\frac{28}{100}}{\frac{1}{2}\times\frac{30}{100}+\frac{1}{2}\times\frac{28}{100}}=\frac{28}{56}=\frac{14}{29}$$

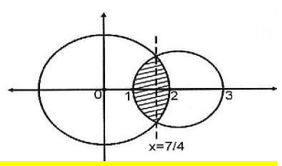
ans : *Yoga increase oxygen combustion in mind and body and meditation increase the concentration of mind. Then we get peace in mind and healthy body. As we know a healthy body can possesses healthy mind.*

Q.26 Draw the rough sketch of the region enclosed between the circles $x^2+y^2=4$ and $(x-2)^2+y^2=1$. Using integration, find the area of the enclosed region .

Ans: Correct figure

Solving $x^2+y^2 = 4$ and $(x-2)^2+y^2 = 1$

we get $x = \frac{7}{4}$




$$\therefore \text{ Required area} = 2\left[\int_{7/4}^{2}\sqrt{4-x^2}\,dx+\int_{1}^{7/4}\sqrt{1-(x-2)^2}\,dx\right] = 2\left[\left[\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}\right]_{\frac{7}{4}}^{2}+\left[\frac{x-2}{2}\sqrt{1-(x-2)^2}+\frac{1}{2}\sin^{-1}\cdot(x-2)\right]_{1}^{\frac{7}{4}}\right]$$

$$= 2\left[\pi-\frac{7}{8}\frac{\sqrt{15}}{4}-2\sin^{-1}\frac{7}{8}+\left(-\frac{1}{8}\cdot\frac{\sqrt{15}}{4}+\frac{1}{2}\sin^{-1}\left(-\frac{1}{4}\right)+\frac{1}{2}\cdot\frac{\pi}{2}\right)\right] = \frac{5\pi}{2}-\frac{\sqrt{15}}{2}-\sin^{-1}\left(\frac{1}{4}\right)-4\sin^{-1}\left(\frac{7}{8}\right)\text{ sq.u}$$

OR

Find the area of the lying circle $x^2+y^2=2ax$ lying above $x$-axis and interior of the



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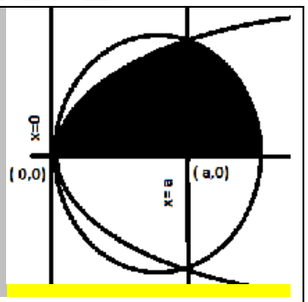



parabola $y^2 = ax$ . Ans.

$$A_1 = \int_0^a \sqrt{ax}\,dx = \frac{2a^2}{3} \;\&\; A_2 = \int_0^a \sqrt{a^2-(x-a)^2}\;0r\int_a^{2a}\sqrt{a^2-(x-a)^2} = \frac{\pi a^2}{4} \therefore A_1 + A_2 = a^2\left(\frac{\pi}{4}+\frac{2}{3}\right)$$

Q.27 Prove that all normal's to the curve $x = a\cos t + at\sin t,\ y = a\sin t - at\cos t$ are at a distance a from the origin. Ans;dy/dx= slope of tangent = tan t ; slope of normal = - cot t then equation of normal : x cos t + y sin t = a & distance from origin is = a

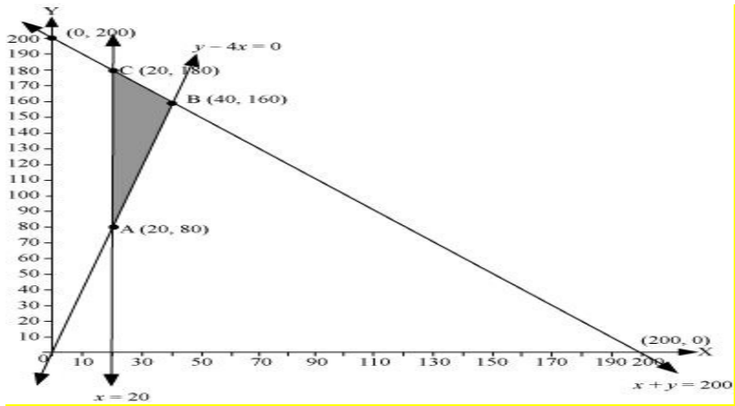
Q.28 Evaluate: $\int_{-1/2}^{1/2} \left| x\cos\frac{\pi x}{2} \right| dx$ . Ans :

$$\int_{-1/2}^{0} -x\cos\frac{\pi x}{2}dx = \frac{1}{\pi\sqrt{2}} + \frac{4}{\pi^2\sqrt{2}} - \frac{4}{\pi^2} + \int_0^{1/2} x\cos\frac{\pi x}{2}dx = \frac{1}{\pi\sqrt{2}} + \frac{4}{\pi^2\sqrt{2}} - \frac{4}{\pi^2} = 2\left(\frac{1}{\pi\sqrt{2}} + \frac{4}{\pi^2\sqrt{2}} - \frac{4}{\pi^2}\right)$$

Q.29 An aero plane can carry a maximum of 200 passengers. A profit of ₹1000 is made on each executive class ticket and a profit of ₹600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximize the profit for the airline. What is the maximum profit? *How one should respect the hard earn money of parents/guardians in a best economical way.Ans:* subject to the constraints,

$$x + y \le 200 \quad ...(2)$$
$$x \ge 20 \quad ...(3)$$
$$y - 4x \ge 0 \quad ...(4)$$
$$x, y \ge 0 \quad ...(5)$$

The feasible region determined by the constraints is as follows.




. The corner points of the feasible region are A (20, 80), B (40, 160), and C (20, 180). The values of $z$ at these corner points are as follows.

| Corner point | z = 1000x + 600y | |
|---|---|---|
| A (20, 80) | 68000 | |
| B (40, 160) | 136000 | → Maximum |
| C (20, 180) | 128000 | |



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| | The maximum value of $z$ is 136000 at (40, 160). Thus, 40 tickets of executive class and 160 tickets of economy class should be sold to maximize the profit and the maximum profit is Rs 136000. Ans : Any justified answer which give clear idea of saving of hard earn money by parents will be awarded |
| | _____x_____ |
| | **USE SOFT WORDS AND HARD ARGUMENTS.** |



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